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# The finite Hilbert transform acting on the Zygmund space *L*log*L*

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**Abstract.** The finite Hilbert transform $T$ is a singular integral operator which maps the Zygmund space $L\log L := L\log L(-1,1)$ continuously into $L^1 := L^1(-1,1)$. By extending the Parseval and Poincaré-Bertrand formulae to this setting, it is possible to establish an inversion result needed for solving the airfoil equation $T(f) = g$ whenever the data function $g$ lies in the range of $T$ within $L^1$ (shown to contain $L(\log L)^2$). Until now this was only known for $g$ belonging to the union of all $L^p$ spaces with $p > 1$. It is established (due to a result of Stein) that $T$ cannot be extended to any domain space beyond $L\log L$ whilst still taking its values in $L^1$, *i.e.*, $T\colon L\log L \to L^1$ is optimally defined.

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