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Periodic solutions to relativistic Kepler problems: a variational approach

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Abstract. We study periodic non-collision solutions to relativistic Kepler problems in the plane. At first, using non-smooth critical point theory, we show that under a general time-periodic external force of gradient type there are two infinite families of T-periodic solutions, parameterized by their winding number around the singularity. The first family is a sequence of local minima, while the second one comes from the application of a new min-max variational principle à la Ghoussoub for non-smooth singular functionals. Secondly, we investigate the minimality of the circular and non-circular periodic solutions of the unforced problem. For this purpose, we combine level estimates of the action functional with an explicit computation of the Morse index of the circular solutions, relying, in turn, on the Conley-Zehnder index of the associated Hamiltonian systems.

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