

Higher-page Hodge theory of compact complex manifolds

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Abstract. On a compact $\partial\bar{\partial}$ -manifold X , one has the Hodge decomposition: the de Rham cohomology groups split into subspaces of pure-type classes as $H_{\text{dR}}^k(X) = \bigoplus_{p+q=k} H^{p,q}(X)$, where the $H^{p,q}(X)$ are canonically isomorphic to the Dolbeault cohomology groups $H_{\bar{\partial}}^{p,q}(X)$. For an arbitrary non-negative integer r , we introduce the class of page- r - $\partial\bar{\partial}$ -manifolds by requiring the analogue of the Hodge decomposition to hold on a compact complex manifold X when the usual Dolbeault cohomology groups $H_{\bar{\partial}}^{p,q}(X)$ are replaced by the spaces $E_{r+1}^{p,q}(X)$ featuring on the $(r+1)$ -st page of the Frölicher spectral sequence of X . The class of page- r - $\partial\bar{\partial}$ -manifolds coincides with the usual class of $\partial\bar{\partial}$ -manifolds when $r=0$ but may grow as r increases. We give two kinds of applications. On the one hand, we give a purely numerical characterisation of the page- r - $\partial\bar{\partial}$ -property in terms of dimensions of various cohomology vector spaces. On the other hand, we obtain several classes of examples, including all complex parallelisable nilmanifolds and certain families of solvmanifolds and Abelian nilmanifolds. Further, there are general results about the behaviour of this new class under standard constructions like blow-ups and deformations.

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