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Distinguished varieties in a family of domains associated with spectral interpolation and operator theory

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Dedicated to Professor Debashish Goswami

Abstract. We characterize all distinguished varieties in the symmetrized polydisc $\mathbb{G}_n$ $(n \geq 2)$ and thus generalize the work [J. Funct. Anal., **266** (2014), 5779–5800] on $\mathbb{G}_2$ by the author and Shalit. We show that a distinguished variety Λ in $\mathbb{G}_n$ is a part of an algebraic curve, which is a set-theoretic complete intersection, and that Λ can be represented by the Taylor joint spectrum of $n-1$ commuting scalar matrices satisfying certain conditions. An n-tuple of commuting Hilbert space operators $(S_1, \dots, S_{n-1}, P)$ for which $\Gamma_n = \overline{\mathbb{G}_n}$ is a spectral set is called a Γ_n-contraction. For every Γ_n-contraction $(S_1, \dots, S_{n-1}, P)$ there is a unique operator tuple $(F_1, \dots, F_{n-1})$, called the $\mathscr{F}_O$-tuple of $(S_1, \dots, S_{n-1}, P)$, satisfying

$$S_i - S_{n-i}^* P = D_P F_i D_P, \quad i = 1, \dots, n-1.$$

We produce a concrete functional model for the pure isometric-operator tuples associated with Γ_n, and by an application of that model we establish that the Γ_n-contractions $(S_1, \dots, S_{n-1}, P)$ and $(S_1^*, \dots, S_{n-1}^*, P^*)$ admit normal $\partial\overline{\Lambda}_\Sigma$-dilations for a unique distinguished variety Λ_Σ in $\mathbb{G}_n$, when Λ_Σ is determined by the $\mathscr{F}_O$-tuple of $(S_1, \dots, S_{n-1}, P)$. Further, we show that the dilation of $(S_1^*, \dots, S_{n-1}^*, P^*)$ is minimal and acts on the minimal unitary dilation space of P^*. Also, we show interplay between the distinguished varieties in $\mathbb{G}_2$ and $\mathbb{G}_3$.

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