Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XXV (2024), 1357-1382

The structure of greedy-type bases in Tsirelson's space and its convexifications

FERNANDO ALBIAC AND JOSÉ L. ANSORENA

Abstract. Tsirelson's space $\mathcal{T}$ made its appearance in Banach space theory in 1974, soon to become one of the most significant counterexamples in the theory. Its structure broke the ideal pattern that analysts had conceived for a generic Banach space, thus giving rise to the era of pathological examples. Since then, many authors have contributed to the study of different aspects of this special space with an eye on better understanding its idiosyncrasies. In this paper we are concerned with the greedy-type basis structure of $\mathcal{T}$, a subject that had not been previously explored in the literature. More specifically, we show that Tsirelson's space and its convexifications $\mathcal{T}^{(p)}$ for $0 < p < \infty$ have uncountably many non-equivalent greedy bases. We also investigate the conditional basis structure of the spaces $\mathcal{T}^{(p)}$ in the range of $0 < p < \infty$ and prove that they have uncountably many non-equivalent conditional almost greedy bases.

Mathematics Subject Classification (2020): 46B03 (primary); 46A16, 46A35, 46A40, 46A45, 46B20, 46B42, 46B45 (secondary).