

Qualitative analysis of optimisation problems with respect to non-constant Robin coefficients

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Abstract. Following recent interest in the qualitative analysis of some optimal control and shape optimisation problems, we provide in this article a detailed study of the optimisation of Robin boundary conditions in PDE constrained calculus of variations. Our main model consists of an elliptic PDE of the form $-\Delta u_\beta = f(x, u_\beta)$ endowed with the Robin boundary conditions $\partial_\nu u_\beta + \beta(x)u_\beta = 0$. The optimisation variable is the function β , which is assumed to take values between 0 and 1 and to have a fixed integral. We consider criteria of two types, beginning with non-energetic ones. In other words, we aim at optimising functionals of the form $\mathcal{J}(\beta) = \int_{\Omega \text{ or } \partial\Omega} j(u_\beta)$. We prove that, depending on the monotonicity of the function j , the optimisers may be of *bang-bang* type (in other words, the optimisers are $\mathbb{1}_\Gamma$ for some measurable subset Γ of $\partial\Omega$) or, on the contrary, that they may only take values strictly between 0 and 1. This has consequences for a related shape optimisation problem, in which one tries to find where on the boundary Neumann ($\partial_\nu u = 0$) and constant Robin conditions ($\partial_\nu u + u = 0$) should be placed in order to optimise criteria. The proofs for this first case rely on new fine oscillatory techniques, used in combination with optimality conditions. Next, we examine the case of compliance-type criteria. For such energetic functionals, we give an in-depth analysis and even some explicit characterisation of the optimal β^* .

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