

## Tachibana-type theorems on complete manifolds

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**Abstract.** We prove that a compact Riemannian manifold of dimension  $m \geq 3$  with harmonic curvature and  $\lfloor \frac{m-1}{2} \rfloor$ -positive curvature operator has constant sectional curvature, extending the classical Tachibana theorem for manifolds with positive curvature operator. The condition of  $\lfloor \frac{m-1}{2} \rfloor$ -positivity originates from recent work of Petersen and Wink, who proved a similar Tachibana-type theorem under the stronger condition that the manifold be Einstein. We show that the same rigidity property holds for complete manifolds assuming either parabolicity, an integral bound on the Weyl tensor or a stronger pointwise positive lower bound on the average of the first  $\lfloor \frac{m-1}{2} \rfloor$  eigenvalues of the curvature operator. For 3-manifolds, we show that positivity of the curvature operator can be relaxed to positivity of the Ricci tensor.

**Mathematics Subject Classification (2020):** 53B20 (primary); 53C20, 53C21, 31C12 (secondary).