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The space of Gauss maps of complete minimal surfaces

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Abstract. The Gauss map of a conformal minimal immersion of an open Riemann surface M into $\mathbb{R}^3$ is a meromorphic function on M. In this paper, we prove that the Gauss map assignment, taking a full conformal minimal immersion $M \to \mathbb{R}^3$ to its Gauss map, is a Serre fibration. We then determine the homotopy type of the space of meromorphic functions on M that are the Gauss map of a complete full conformal minimal immersion, and show that it is the same as the homotopy type of the space of all continuous maps from M to the 2-sphere. We obtain analogous results for the generalised Gauss map of conformal minimal immersions $M \to \mathbb{R}^n$ for arbitrary $n \geq 3$.

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