

Finite point blowup for the critical generalized Korteweg-de Vries equation

YVAN MARTEL AND DIDIER PILOD

Abstract. In the last twenty years, there have been significant advances in the study of the blow-up phenomenon for the critical generalized Korteweg-de Vries equation, including the determination of sufficient conditions for blowup, the stability of blowup in a refined topology and the classification of minimal mass blowup. Exotic blow-up solutions with a continuum of blow-up rates and multi-point blow-up solutions were also constructed. However, all these results, as well as numerical simulations, involve the bubbling of a solitary wave going to infinity at the blow-up time, which means that the blow-up dynamics and the residue are eventually uncoupled. Even at the formal level, the question whether blowup at a finite point could occur for this equation remained open. In this article, we answer this question by constructing solutions that blow up in finite time under the form of a single bubble concentrating the ground state at a finite point with an unforeseen blow-up rate. The fact that we find a blow-up rate intermediate between the self-similar rate and other rates previously known also reopens the question of which blow-up rates are actually possible for this equation.

Mathematics Subject Classification (2020): 35Q53 (primary); 35B44, 35Q51 (secondary).