Unextendable intrinsic Lipschitz curves

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Abstract. In the setting of Carnot groups, we exhibit examples of intrinsic Lipschitz curves of positive $\mathcal{H}^1$-measure that intersect every connected intrinsic Lipschitz curve in an $\mathcal{H}^1$-negligible set. As a consequence one deduces that such curves cannot be extended to connected intrinsic Lipschitz curves.

The examples are constructed in the Engel group and in the free Carnot group of step 3 and rank 2. While the failure of the Lipschitz extension property was already known for some pairs of Carnot groups, ours is the first example of the analogous phenomenon for intrinsic Lipschitz graphs. This is in sharp contrast with the Euclidean case.

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