

Harmonic intrinsic graphs in the Heisenberg group

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Abstract. Minimal surfaces in \mathbb{R}^n can be locally approximated by graphs of harmonic functions, *i.e.*, functions that are critical points of the Dirichlet energy, but no analogous theorem is known for H -minimal surfaces in the three-dimensional Heisenberg group \mathbb{H} , which are known to have singularities. In this paper, we introduce a definition of intrinsic Dirichlet energy for surfaces in \mathbb{H} and study the critical points of this energy, which we call contact harmonic graphs. Nearly flat regions of H -minimal surfaces can often be approximated by such graphs. We give a calibration condition for an intrinsic Lipschitz graph to be energy-minimizing, construct energy-minimizing graphs with a variety of singularities, and prove a first variation formula for the energy of intrinsic Lipschitz graphs and piecewise smooth intrinsic graphs.

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