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Delsarte's extremal problem and packing on locally compact Abelian groups

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Abstract. Let G be a locally compact Abelian group, and let Ω_+, Ω_- be two open sets in G. We investigate the constant

$$\mathcal{C}(\Omega_+, \Omega_-) = \sup \left\{ \int_G f : f \in \mathcal{F}(\Omega_+, \Omega_-) \right\},$$

where $\mathcal{F}(\Omega_+, \Omega_-)$ is the class of positive-definite functions f on G such that $f(0) = 1$, the positive part f_+ of f is supported in Ω_+, and its negative part f_- is supported in Ω_-. In the case when $\Omega_+ = \Omega_- =: \Omega$, the problem is exactly the so-called Turán problem for the set Ω. When $\Omega_- = G$, *i.e.*, there is a restriction only on the set of positivity of f, we obtain the Delsarte problem. The Delsarte problem in $\mathbb{R}^d$ is the sharpest Fourier analytic tool to study packing density by translates of a given "master copy" set, which was studied first in connection with packing densities of Euclidean balls.

We give an upper estimate of the constant $\mathcal{C}(\Omega_+, \Omega_-)$ in the situation when the set Ω_+ satisfies a certain packing type condition. This estimate is given in terms of the asymptotic uniform upper density of sets in locally compact Abelian groups.

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