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Ground state energy threshold and blow-up for NLS with competing nonlinearities

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Abstract. We consider a 3D nonlinear Schrödinger equation with combined nonlinearities, where the leading term is an intercritical focusing power-type nonlinearity, and the perturbation is given by a power-type defocusing one. We completely answer the question whether the ground state energy, which is a threshold between global existence and formation of singularities, is achieved. For any prescribed mass, for mass-supercritical or mass-critical defocusing perturbations, the ground state energy is achieved by a radially symmetric and decreasing solution to the associated stationary equation. For mass-subcritical perturbations, we show the existence of a critical prescribed mass, precisely the mass of the unique, static, positive solution to the associated elliptic equation, such that the ground state energy is achieved for any mass equal or smaller than the critical one. Moreover we prove that the ground state energy is not achieved for any mass larger than the critical one. As a byproduct of the variational characterization of the ground state energy, we prove the existence of blowing-up solutions in finite-time for any energy below the ground state energy threshold.

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