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Existence and local uniqueness of normalized peak solutions for a Schrödinger-Newton system

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Abstract. In this paper we investigate the existence and local uniqueness of normalized peak solutions for a Schrödinger-Newton system under the assumption that the trapping potential is degenerate and has nonisolated critical points.

First we investigate the existence and local uniqueness of normalized single-peak solutions for the Schrödinger-Newton system. Precisely, we give an exact description of the chemical potential and the attractive interaction. Then we apply the finite-dimensional reduction method to obtain the existence of single-peak solutions. Furthermore, using various local Pohozaev identities, blow-up analysis and the maximum principle, we prove the local uniqueness of single-peak solutions by a precise analysis of the concentrated points and the Lagrange multiplier. Finally, we also prove the nonexistence of multi-peak solutions for the Schrödinger-Newton system, which is markedly different from the corresponding Schrödinger equation. This difference entails the existence of a nonlocal term.

The main difficulties come from the estimates on the Lagrange multiplier, the different degenerate rates along different directions at the critical point of the potential function and some complicated estimates involving the nonlocal term. To our best knowledge, this may be the first paper to study the existence and local uniqueness of solutions with prescribed L^2-norm for the Schrödinger-Newton system.

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