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Quantitative particle approximation of nonlinear Fokker-Planck equations with singular kernel

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Abstract. In this work, we study the convergence of the empirical measure of moderately interacting particle systems with singular interaction kernels. First, we prove quantitative convergence of the time marginals of the empirical measure of particle positions towards the solution of the limiting nonlinear Fokker-Planck equation. Second, we prove the well-posedness for the McKean-Vlasov SDE involving such singular kernels and the convergence of the empirical measure towards it (propagation of chaos).

Our results only require very weak regularity on the interaction kernel, which permits to treat models for which the mean field particle system is not known to be well-defined. For instance, this includes attractive kernels such as Riesz and Keller-Segel kernels in arbitrary dimension. For some of these important examples, this is the first time that a quantitative approximation of the PDE is obtained by means of a stochastic particle system. In particular, this convergence still holds (locally in time) for PDEs exhibiting a blow-up in finite time. The proofs are based on a semigroup approach combined with a fine analysis of the regularity of infinite-dimensional stochastic convolution integrals.

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