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Liouville theorems for infinity Laplacian with gradient and KPP type equation

ANUP BISWAS AND HOANG-HUNG VO[1]

Abstract. In this paper, we prove new Liouville type results for a nonlinear equation involving infinity Laplacian with gradient, of the form

$$\Delta_\infty^\gamma u + q(x) \cdot \nabla u |\nabla u|^{2-\gamma} + f(x,u) = 0 \quad \text{in } \mathbb{R}^d,$$

where $\gamma \in [0,2]$ and Δ_∞^γ is a $(3-\gamma)$-homogeneous operator associated with the infinity Laplacian. Under the assumption that $\liminf_{|x|\to\infty} \lim_{s\to 0} f(x,s)/s^{3-\gamma} > 0$ and that q is a continuous function vanishing at infinity, we construct a positive bounded solution to the equation, and if $f(x,s)/s^{3-\gamma}$ is decreasing in s, we further obtain its uniqueness by improving a sliding method for infinity Laplacian operators with nonlinear gradient. Otherwise, if $\limsup_{|x|\to\infty} \sup_{[\delta_1,\delta_2]} f(x,s) < 0$, then under some suitable additional conditions a nonexistence result holds. To this aim, we develop novel techniques to overcome the difficulties stemming from the degeneracy of infinity Laplacian and nonlinearity of the gradient term. Our approach is based on a new regularity result, a strong maximum principle, and a Hopf lemma for infinity Laplacian involving gradient and potential. We also construct some examples to illustrate our results. We further investigate some deeper qualitative properties of the principal eigenvalue of the corresponding nonlinear operator

$$\Delta_\infty^\gamma u + q(x) \cdot \nabla u |\nabla u|^{2-\gamma} + c(x) u^{3-\gamma},$$

with Dirichlet boundary condition in smooth bounded domains, which may be of independent interest. The results obtained here could be considered as sharp extensions of the Liouville type results obtained in [1, 2, 11, 24, 48, 52].

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