

On the Kodaira-Spencer map of Abelian schemes

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Abstract. Let A be an Abelian scheme over a smooth affine complex variety S , Ω_A the \mathcal{O}_S -module of 1-forms of the first kind on A , $\mathcal{D}_S\Omega_A$ the \mathcal{D}_S -module spanned by Ω_A in the first algebraic De Rham cohomology module, and $\theta_\partial : \Omega_A \rightarrow \mathcal{D}_S\Omega_A/\Omega_A$ the Kodaira-Spencer map attached to a tangent vector field ∂ on S . We compare the rank of $\mathcal{D}_S\Omega_A/\Omega_A$ to the maximal rank of θ_∂ when ∂ varies: we show that both ranks do not change when one passes to the “modular case”, *i.e.* when one replaces S by the smallest weakly special subvariety of \mathcal{A}_g containing the image of S (assuming, as one may up to isogeny, that A/S is principally polarized); we then analyse the “modular case” and deduce, for instance, that for any Abelian pencil of relative dimension g with Zariski-dense monodromy in Sp_{2g} , the derivative with respect to a parameter of a non zero Abelian integral of the first kind is never of the first kind.

Mathematics Subject Classification (2010): 14K20 (primary); 14G35, 32G20 (secondary).