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Asymptotic behavior of stochastic currents under large deviation scaling with mean field interaction and vanishing noise

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Abstract. We study the large deviation behavior of a system of diffusing particles with a mean field interaction, described through a collection of stochastic differential equations, in which each particle is driven by a vanishing independent Brownian noise. An important object in the description of the asymptotic behavior, as the number of particles approaches infinity and the noise intensity approaches zero, is the stochastic current associated with the interacting particle system in the sense of Flandoli *et al.* (2005). We establish a joint large deviation principle (LDP) for the path empirical measure for the particle system and the associated stochastic currents in the simultaneous large particle and small noise limit. Our work extends recent results of Orrieri (2018), in which the diffusion coefficient is taken to be the identity, to a setting of a state dependent and possibly degenerate noise with the mean field interaction influencing both the drift and diffusion coefficients, and allowing for a stronger topology on the space of stochastic currents in the LDP. Proof techniques differ from those used by Orrieri (2018) and rely on methods from stochastic control, theory of weak convergence, and representation formulas for Laplace functionals of Brownian motions.

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