

On Sirakov's open problem and related topics

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Abstract. In the present paper, we make some progress on the Sirakov's open problem (Comm. Math. Phys., 2007) about the existence of nontrivial nonnegative solution to the coupled nonlinear system

$$\begin{cases} -\Delta u + \lambda u = \mu_1 u^3 + \beta uv^2 & \text{in } \mathbb{R}^N \\ -\Delta v + v = \mu_2 v^3 + \beta u^2 v & \text{in } \mathbb{R}^N \\ u, v > 0 & \text{in } \mathbb{R}^N \quad N \leq 3. \end{cases}$$

We also study some other properties for related questions, such as the uniqueness of the ground state solution, the asymptotic behavior of the least energy solution, nonexistence of the positive solution and the multiplicity of positive solutions, etc.

Mathematics Subject Classification (2020): 35J50 (primary); 35J15, 35J60 (secondary).