

Transport Equations with Partially BV Velocities

NICOLAS LERNER

Abstract. We prove the uniqueness of weak solutions for the Cauchy problem for a class of transport equations whose velocities are partially with bounded variation. Our result deals with the initial value problem $\partial_t u + Xu = f$, $u|_{t=0} = g$, where X is the vector field

$$a_1(x_1) \cdot \partial_{x_1} + a_2(x_1, x_2) \cdot \partial_{x_2}, \quad a_1 \in BV(\mathbb{R}_{x_1}^{N_1}), \quad a_2 \in L_{x_1}^1(BV(\mathbb{R}_{x_2}^{N_2})),$$

with a boundedness condition on the divergence of each vector field a_1, a_2 . This model was studied in the paper [LL] with a $W^{1,1}$ regularity assumption replacing our BV hypothesis. This settles partly a question raised in the paper [Am]. We examine the details of the argument of [Am] and we combine some consequences of the Alberti rank-one structure theorem for BV vector fields with a regularization procedure. Our regularization kernel is not restricted to be a convolution and is introduced as an unknown function. Our method amounts to commute a pseudo-differential operator with a BV function.

Mathematics Subject Classification (2000): 35F05 (primary); 34A12, 26A45 (secondary).