

Spines of minimal length

BRUNO MARTELLI, MATTEO NOVAGA, ALESSANDRA PLUDA
AND STEFANO RIOLO

Abstract. In this paper we raise the question whether every closed Riemannian manifold has a spine of minimal area, and we answer it affirmatively in the surface case. On constant curvature surfaces we introduce the *spine systole*, a continuous real function on moduli space that measures the minimal length of a spine in each surface. We show that the spine systole is a proper function and has its global minima precisely on the extremal surfaces (those containing the biggest possible discs).

We also study *minimal spines*, which are critical points for the length functional. We completely classify minimal spines on flat tori, proving that the number of them is a proper function on moduli space. We also show that the number of minimal spines of uniformly bounded length is finite on hyperbolic surfaces.

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