Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XVII (2017), 969-1029

Dispersive estimates with loss of derivatives via the heat semigroup and the wave operator

FRÉDÉRIC BERNICOT AND VALENTIN SAMOYEAU

Abstract. In this paper our aim is to give a general (possibly compact or non-compact) analog of the Strichartz inequalities with loss of derivatives, obtained by Burq, Gérard and Tzvetkov [21] and Staffilani and Tataru [56]. Moreover we present a new approach, relying only on the heat semigroup, in order to understand the analytic connection between the heat semigroup and the unitary Schrödinger group (both related to a same self-adjoint operator). One of the novelties is to forget the endpoint L^1-L^∞ dispersive estimate and to look for a weaker H^1-BMO estimate (Hardy and BMO spaces both adapted to the heat semigroup). This new point of view allows us to give a general framework (infinite metric spaces, Riemannian manifolds with rough metric, manifolds with boundary, ...) where Strichartz inequalities with loss of derivatives can be reduced to microlocalized L^2-L^2 dispersive properties. We also use the link between the wave propagator and the unitary Schrödinger group to prove how short-time dispersion for waves implies dispersion for the Schrödinger group.

Mathematics Subject Classification (2010): 35B30 (primary); 42B37, 47D03, 47D06 (secondary).