On the *b*-ary expansions of $\log\left(1 + \frac{1}{a}\right)$ and e

YANN BUGEAUD AND DONG HAN KIM

Abstract. Let $b \geq 2$ be an integer and ξ be an irrational real number. We prove that, if the irrationality exponent of ξ is equal to 2 or slightly greater than 2, then the b-ary expansion of ξ cannot be "too simple", in a suitable sense. Our result applies to, among other classical numbers, to badly approximable numbers, non-zero rational powers of e, and $\log(1+\frac{1}{a})$, provided that the integer a is sufficiently large. It establishes an unexpected connection between the irrationality exponent of a real number and its b-ary expansion.