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Characterizations of signed measures in the dual of BV and related isometric isomorphisms

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This paper is dedicated to William P. Ziemer on the occasion of his 81st birthday

Abstract. We characterize all (signed) measures in $BV_{\frac{n}{n-1}}(\mathbb{R}^n)^*$, where $BV_{\frac{n}{n-1}}(\mathbb{R}^n)$ is defined as the space of all functions u in $L^{\frac{n}{n-1}}(\mathbb{R}^n)$ such that Du is a finite vector-valued measure. We also show that $BV_{\frac{n}{n-1}}(\mathbb{R}^n)^*$ and $BV(\mathbb{R}^n)^*$ are isometrically isomorphic, where $BV(\mathbb{R}^n)$ is defined as the space of all functions u in $L^1(\mathbb{R}^n)$ such that Du is a finite vector-valued measure. As a consequence of our characterizations, an old issue raised in Meyers-Ziemer [19] is resolved by constructing a locally integrable function f such that f belongs to $BV(\mathbb{R}^n)^*$ but $|f|$ does not. Moreover, we show that the measures in $BV_{\frac{n}{n-1}}(\mathbb{R}^n)^*$ coincide with the measures in $\dot{W}^{1,1}(\mathbb{R}^n)^*$, the dual of the homogeneous Sobolev space $\dot{W}^{1,1}(\mathbb{R}^n)$, in the sense of isometric isomorphism. For a bounded open set Ω with Lipschitz boundary, we characterize the measures in the dual space $BV_0(\Omega)^*$. One of the goals of this paper is to make precise the definition of $BV_0(\Omega)$, which is the space of functions of bounded variation with zero trace on the boundary of Ω. We show that the measures in $BV_0(\Omega)^*$ coincide with the measures in $W_0^{1,1}(\Omega)^*$. Finally, the class of finite measures in $BV(\Omega)^*$ is also characterized.

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