Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XVII (2017), 205-230

Certain sets over function fields are polynomial families

NGUYEN NGOC DONG QUAN

Abstract. In 1938, Skolem conjectured that $\mathbf{SL}_n(\mathbb{Z})$ is not a polynomial family for any $n \geq 2$. Carter and Keller disproved Skolem's conjecture for all $n \geq 3$ by proving that $\mathbf{SL}_n(\mathbb{Z})$ is boundedly generated by the elementary matrices, and hence a polynomial family for any $n \geq 3$. Only recently, Vaserstein refuted Skolem's conjecture completely by showing that $\mathbf{SL}_2(\mathbb{Z})$ is a polynomial family. An immediate consequence of Vaserstein's theorem also implies that $\mathbf{SL}_n(\mathbb{Z})$ is a polynomial family for any $n \geq 3$. In this paper, we prove a function field analogue of Vaserstein's theorem: that is, if $\mathbf{A}$ is the ring of polynomials over a finite field of odd characteristic, then $\mathbf{SL}_2(\mathbf{A})$ is a polynomial family in 52 variables. A consequence of our main result also implies that $\mathbf{SL}_n(\mathbf{A})$ is a polynomial family for any $n \geq 3$.

Mathematics Subject Classification (2010): 11D72 (primary); 11R58 (secondary).