Failure of the chain rule for the divergence of bounded vector fields

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Abstract. We provide a vast class of counterexamples to the chain rule for the divergence of bounded vector fields in three space dimensions. Our convex integration approach allows us to produce renormalization defects of various kinds, which in a sense quantify the breakdown of the chain rule. For instance, we can construct defects which are absolutely continuous with respect to the Lebesgue measure, or defects which are not even measures.