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Bergman kernel and projection on the unbounded Diederich–Fornæss worm domain

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Abstract. In this paper we study the Bergman kernel and projection on the unbounded worm domain

$$\mathcal{W}_\infty = \left\{ (z_1, z_2) \in \mathbb{C}^2 : \left| z_1 - e^{i \log |z_2|^2} \right|^2 < 1 \ \text{ for } \ z_2 \neq 0 \right\} .$$

We first show that the Bergman space of $\mathcal{W}_\infty$ is infinite dimensional. Then we study the Bergman kernel K and the Bergman projection $\mathcal{P}$ for $\mathcal{W}_\infty$. We prove that $K(z, w)$ extends holomorphically in z (and antiholomorphically in w) near each point of the boundary except for a specific subset that we study in detail. By means of an appropriate asymptotic expansion for K, we prove that the Bergman projection $\mathcal{P} : W^s \not\to W^s$ if $s > 0$ and $\mathcal{P} : L^p \not\to L^p$ if $p \neq 2$, where W^s and L^p denote the classic Sobolev space, and the Lebesgue space, respectively, on $\mathcal{W}_\infty$.

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