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# Infinitely many sign-changing and semi-nodal solutions for a nonlinear Schrödinger system

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**Abstract.** We study the following coupled Schrödinger equations which have appeared as several models from mathematical physics:

$$\begin{cases} -\Delta u_1 + \lambda_1 u_1 = \mu_1 u_1^3 + \beta u_1 u_2^2 & x \in \Omega \\ -\Delta u_2 + \lambda_2 u_2 = \mu_2 u_2^3 + \beta u_1^2 u_2 & x \in \Omega \\ u_1 = u_2 = 0 & \text{on } \partial\Omega. \end{cases}$$

Here $\Omega$ is a smooth bounded domain in $\mathbb{R}^N (N = 2, 3)$ or $\Omega = \mathbb{R}^N$, $\lambda_1$, $\lambda_2$, $\mu_1$, $\mu_2$ are all positive constants and the coupling constant $\beta < 0$. We show that this system has infinitely many sign-changing solutions. We also obtain infinitely many semi-nodal solutions in the following sense: one component changes sign and the other one is positive. The crucial idea of our proof, which has never been used for this system before, is to study a new problem with two constraints. Finally, when $\Omega$ is a bounded domain, we show that this system has a least energy sign-changing solution, both two components of which have exactly two nodal domains, and we also study the asymptotic behavior of solutions as $\beta \to -\infty$ and phase separation is expected.

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