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General optimal L^p-Nash inequalities on Riemannian manifolds

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Abstract. Let (M, g) be a smooth compact Riemannian manifold of dimension $2 \leq n$, let $1 < p$ and $1 \leq q < p$. In this paper, we establish the validity of the optimal Nash inequality

$$\left(\int_M |u|^p \, dv_g\right)^{\frac{\tau}{\theta p}} \leq \left(A_{\text{opt}} \left(\int_M |\nabla_g u|^p \, dv_g\right)^{\frac{\tau}{p}} + B_{\text{opt}} \left(\int_M |u|^p \, dv_g\right)^{\frac{\tau}{p}}\right) \left(\int_M |u|^q \, dv_g\right)^{\frac{\tau(1-\theta)}{\theta q}},$$

and the existence of extremal functions for this optimal inequality.

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