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Bergman-harmonic maps of balls

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Abstract. We study Bergman-harmonic maps between balls $\Phi : \mathbb{B}_n \to \mathbb{B}_N$ extending of class either C^2 or $\mathfrak{M}^1$ to the boundary of $\mathbb{B}_n$. For every holomorphic (anti-holomorphic) map $\Phi : \mathbb{B}_n \to \mathbb{B}_N$ extending smoothly to the boundary and every smooth homotopy $H : \Phi \simeq \Psi$ we prove a Lichnerowicz-type (*cf.* [28]) result, *i.e.*, we show that $E_{\Omega_\epsilon}(\Psi) \geq E_{\Omega_\epsilon}(\Phi) + O(\epsilon^{-n+1})$. When Φ is proper, Bergman-harmonic, and C^2 up to the boundary, the boundary values map $\phi : S^{2n-1} \to S^{2N-1}$ is shown to satisfy a compatibility system similar to the tangential Cauchy-Riemann equations on S^{2n-1} (and satisfied by the boundary values of any proper holomorphic map). For every weakly Bergman-harmonic map $\Phi \in W^1(\mathbb{B}_n, \mathbb{B}_N)$ admitting Sobolev boundary values $\phi \in \mathfrak{M}^1(S^{2n-1}, \mathbb{B}_N)$ in the sense of [6], the boundary values ϕ are shown to be a weakly subelliptic harmonic map of (S^{2n-1}, η) into $(\mathbb{B}_N, h)$, provided that $\Phi^{-1}\nabla^h$ stays bounded at the boundary of $\mathbb{B}_n$ and ϕ has vanishing weak normal derivatives.

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