# Sobolev extension property for tree-shaped domains with self-contacting fractal boundary

THIBAUT DEHEUVELS

**Abstract.** In this paper, we investigate the existence of $W^{1,p}$-extension operators for a class of bidimensional ramified domains with a self-similar fractal boundary previously studied by Mandelbrot and Frame. When the fractal boundary has no self-contact, the domains have the $(\epsilon, \delta)$-property, and the extension results of Jones imply that there exist such extension operators for all $1 \leqslant p \leqslant \infty$. In the case where the fractal boundary self-intersects, this result does not hold. In this work we construct extension operators for $1 < p < p^{\star}$, where $p^{\star}$ depends only on the dimension of the self-intersection of the boundary. The construction of the extension operators is based on a Haar wavelet decomposition on the fractal part of the boundary. It relies mainly on the self-similar properties of the domain. The result is sharp in the sense that $W^{1,p}$-extension operators fail to exist when $p > p^{\star}$.

**Mathematics Subject Classification (2010):** 46E35(primary); 28A80 (secondary).