

The Lie algebra generated by locally nilpotent derivations on a Danielewski surface

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Abstract. We give a full description of the Lie algebra generated by locally nilpotent derivations (shortly LNDs) on smooth Danielewski surfaces D_p given by $xy = p(z)$. In case $\deg(p) \geq 3$ it turns out that it is not equal to the whole Lie algebra $\text{VF}_{\text{alg}}^\omega(D_p)$ of volume-preserving algebraic vector fields, thus answering a question posed by Lind and the first author. We also show an algebraic volume density property (shortly AVDP) for a certain homology plane (a homogeneous space of the form $SL_2(\mathbb{C})/N$, where N is the normalizer of the maximal torus) and a related example.

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