

Failure of the local-to-global property for $CD(K, N)$ spaces

TAPIO RAJALA

Abstract. Given any $K \in \mathbb{R}$ and $N \in [1, \infty]$ we show that there exists a compact geodesic metric measure space satisfying locally the $CD(0, 4)$ condition but failing to satisfy $CD(K, N)$ globally. The space with this property is a suitable non-convex subset of \mathbb{R}^2 equipped with the l^∞ -norm and the Lebesgue measure. Combining many such spaces gives a (non-compact) complete geodesic metric measure space satisfying $CD(0, 4)$ locally but failing to satisfy $CD(K, N)$ globally for every K and N .

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