Multiple zeta values, Padé approximation and Vasilyev's conjecture

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Abstract. Sorokin gave in 1996 a new proof that π is transcendental. It is based on a simultaneous Padé approximation problem involving certain multiple polylogarithms, which evaluated at the point 1 are multiple zeta values equal to powers of π. In this paper we construct a Padé approximation problem of the same flavour, and prove that it has a unique solution up to proportionality. At the point 1, this provides a rational linear combination of 1 and multiple zeta values in an extended sense that turn out to be values of the Riemann zeta function at odd integers. As an application, we obtain a new proof of Vasilyev's conjecture for any odd weight, which concerns the explicit evaluation of certain hypergeometric multiple integrals, first proved by Zudilin in 2003.