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Harmonic Bergman spaces, the Poisson equation and the dual of Hardy-type spaces on certain noncompact manifolds

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Abstract. In this paper we consider a complete connected noncompact Riemannian manifold M with bounded geometry and spectral gap. We realize the dual space $Y^k(M)$ of the Hardy-type space $X^k(M)$, introduced in a previous paper of the authors, as the class of all locally square integrable functions satisfying suitable BMO-like conditions, where the role of the constants is played by the space of global k-quasi-harmonic functions. Furthermore we prove that $Y^k(M)$ is also the dual of the space $X^k_{\mathrm{fin}}(M)$ of finite linear combination of X^k-atoms. As a consequence, if Z is a Banach space and T is a Z-valued linear operator defined on $X^k_{\mathrm{fin}}(M)$, then T extends to a bounded operator from $X^k(M)$ to Z if and only if it is uniformly bounded on X^k-atoms. To obtain these results we prove the global solvability of the generalized Poisson equation $\mathscr{L}^k u = f$ with $f \in L^2_{\mathrm{loc}}(M)$ and we study some properties of generalized Bergman spaces of harmonic functions on geodesic balls.

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