

## Control theorems for $\ell$ -adic Lie extensions of global function fields

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**Abstract.** Let  $F$  be a global function field of characteristic  $p > 0$ ,  $K/F$  an  $\ell$ -adic Lie extension unramified outside a finite set of places  $S$  and  $A/F$  an abelian variety. We study  $\text{Sel}_A(K)_\ell^\vee$  (the Pontrjagin dual of the Selmer group) and (under some mild hypotheses) prove that it is a finitely generated  $\mathbb{Z}_\ell[[\text{Gal}(K/F)]]$ -module via generalizations of Mazur's Control Theorem. If  $\text{Gal}(K/F)$  has no elements of order  $\ell$  and contains a closed normal subgroup  $H$  such that  $\text{Gal}(K/F)/H \simeq \mathbb{Z}_\ell$ , we are able to give sufficient conditions for  $\text{Sel}_A(K)_\ell^\vee$  to be finitely generated as  $\mathbb{Z}_\ell[[H]]$ -module and, consequently, a torsion  $\mathbb{Z}_\ell[[\text{Gal}(K/F)]]$ -module. We deal with both cases  $\ell \neq p$  and  $\ell = p$ .

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