

Blow up for the critical gKdV equation III: exotic regimes

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Abstract. We consider the blow-up problem in H^1 for the L^2 critical generalized Korteweg–de Vries (gKdV) equation, as a continuation of [38, 39]. We know from [38] that the unique and stable blow-up rate for H^1 solutions close to the solitons with strong decay on the right is

$$\|u_x(t)\|_{L^2} \sim \frac{1}{T-t} \quad \text{as } t \uparrow T < +\infty.$$

In this paper we construct non-generic blow-up regimes in H^1 by considering initial data with explicit slow decay on the right in space. We obtain finite time blow-up solutions with speed

$$\|u_x(t)\|_{L^2} \sim \frac{1}{(T-t)^\nu} \quad \text{as } t \uparrow T < +\infty, \quad \nu > \frac{11}{13},$$

as well as global in time growing up solutions with exponential growth

$$\|u_x(t)\|_{L^2} \sim e^t \quad \text{as } t \rightarrow +\infty,$$

or growth of any power

$$\|u_x(t)\|_{L^2} \sim t^\nu \quad \text{as } t \rightarrow +\infty, \quad \nu > 0.$$

These solutions can be taken with initial data arbitrarily close in H^1 to the ground state solitary wave.

Mathematics Subject Classification (2010): 35Q53 (primary); 35Q51, 35B44, 35B35 (secondary).