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Existence of strong solutions for quasi-static evolution in brittle fracture

JEAN-FRANÇOIS BABADJIAN AND ALESSANDRO GIACOMINI

Abstract. This paper is devoted to prove the existence of strong solutions for a brittle fracture model of quasi-static crack propagation in the two dimensional antiplane setting. As usual, the time continuous evolution is obtained as the limit of a discrete in time evolution by letting the time step tend to zero. The analysis rests on a density lower bound estimate for quasi-minimizers of Mumford-Shah type functionals, under a homogeneous Dirichlet boundary condition on a part of the boundary. In contrast with the previous results, since boundary cracks may be obtained as limits of interior cracks, such a density lower bound has to be established also on balls centered inside the domain but possibly intersecting the Dirichlet boundary. Thanks to a 2D geometrical argument, the discrete in time crack turns out to satisfy a uniform density lower bound which can pass to the limit, leading to the closedness of the continuous in time crack. We also establish better convergence properties of the discrete in time displacement/crack pair towards its time continuous counterpart.

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