Vanishing of special values and central derivatives in Hida families

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Abstract. The theme of this work is the study of the Nekovář-Selmer group $\widetilde{H}^1_f(K, \mathbb{T}^\dagger)$ attached to a twisted Hida family $\mathbb{T}^\dagger$ of Galois representations and a quadratic number field K. The results that we obtain have the following shape: if a twisted L-function of a suitable modular form in the Hida family has order of vanishing $r \leq 1$ at the central critical point then the rank of $\widetilde{H}^1_f(K, \mathbb{T}^\dagger)$ as a module over a certain local Hida-Hecke algebra is equal to r. Under the above assumption, we also show that infinitely many twisted L-functions of modular forms in the Hida family have the same order of vanishing at the central critical point. Our theorems extend to more general arithmetic situations results obtained by Howard when K is an imaginary quadratic field and all the primes dividing the tame level of the Hida family split in K.