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Semigroups generated by elliptic operators in non-divergence form on $C_0(\Omega)$

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Abstract. Let $\Omega \subset \mathbb{R}^n$ be a bounded open set satisfying the uniform exterior cone condition. Let $\mathcal{A}$ be a uniformly elliptic operator given by

$$\mathcal{A}u = \sum_{i,j=1}^{n} a_{ij}\partial_{ij}u + \sum_{j=1}^{n} b_j\partial_j u + cu$$

where

$$a_{ji} = a_{ij} \in C(\bar{\Omega}) \text{ and } b_j, c \in L^\infty(\Omega), c \leq 0 .$$

We show that the realization A_0 of $\mathcal{A}$ in

$$C_0(\Omega) := \{u \in C(\bar{\Omega}) : u_{|\partial\Omega} = 0\}$$

given by

$$\begin{aligned} D(A_0) &:= \{u \in C_0(\Omega) \cap W^{2,n}_{\mathrm{loc}}(\Omega) : \mathcal{A}u \in C_0(\Omega)\} \\ A_0 u &:= \mathcal{A}u \end{aligned}$$

generates a bounded holomorphic C_0-semigroup on $C_0(\Omega)$. The result is in particular true if Ω is a Lipschitz domain. So far the best known result seems to be the case where Ω has C^2-boundary [12, Section 3.1.5]. We also study the elliptic problem

$$\begin{aligned} -\mathcal{A}u &= f \\ u_{|\partial\Omega} &= g . \end{aligned}$$

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