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On compactness in the Trudinger-Moser inequality

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Abstract. We show that the Moser functional $J(u) = \int_\Omega (e^{4\pi u^2} - 1)\,\mathrm{d}x$ on the set $\mathcal{B} = \{u \in H_0^1(\Omega) : \|\nabla u\|_2 \leq 1\}$, where $\Omega \subset \mathbb{R}^2$ is a bounded domain, fails to be weakly continuous only in the following exceptional case. Define $g_s w(r) = s^{-\frac{1}{2}} w(r^s)$ for $s > 0$. If $u_k \rightharpoonup u$ in $\mathcal{B}$ while $\liminf J(u_k) > J(u)$, then, with some $s_k \to 0$,

$$u_k = g_{s_k}\left[(2\pi)^{-\frac{1}{2}} \min\left\{1, \log\frac{1}{|x|}\right\}\right],$$

up to translations and up to a remainder vanishing in the Sobolev norm. In other words, the weak continuity fails only on translations of concentrating Moser functions. The proof is based on a profile decomposition similar to that of Solimini [16], but with different concentration operators, pertinent to the two-dimensional case.

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