Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XII (2013), 833-862

Reduction of critical mass in a chemotaxis system by external application of a chemoattractant

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Abstract. In this paper we study non-negative radially symmetric solutions of the parabolic-elliptic Keller-Segel system

$$\begin{cases} u_t = \Delta u - \nabla \cdot (u \nabla v), & x \in \mathbb{R}^2,\ t > 0, \\ 0 = \Delta v + u + f_0 \cdot \delta(x), & x \in \mathbb{R}^2,\ t > 0, \end{cases} \tag{$\star$}$$

where $f_0 > 0$ and δ is the Dirac distribution. This system describes the chemotactic movement of cells under the additional circumstance that an external application of a chemoattractant at a distinguished point is introduced.

It is known that without such an external source the number 8π plays the role of a *critical mass* in ($\star$), in the sense that if the total mass $\mu := \int_{\mathbb{R}^2} u_0$ of the cells exceeds 8π then the solutions may blow up within finite time and collapse into a Dirac-type singularity, and that this does not occur when $\mu < 8\pi$.

The present paper shows that this critical number is reduced to $8\pi - 2f_0$ by an application of the signal substance in the above way. Indeed, it is proved that whenever $f_0 > 0$ and $u_0 \not\equiv 0$, a measure-valued global-in-time weak solution can be constructed which blows up at $x = 0$ immediately. Now if $\mu < 8\pi - 2f_0$ then this solution satisfies $u(x,t) \le C(\tau)|x|^{-\frac{f_0}{2\pi}}$ for $t > \tau > 0$ and $|x| < 1$ and hence does not blow up in $L^p_{loc}(\mathbb{R}^2)$ for any $1 \le p < 4\pi/f_0$. On the other hand, if $\mu > 8\pi - f_0$ then the mass will asymptotically completely concentrate at the origin, that is, $u(\cdot,t)$ converges to $\mu \cdot \delta$ as $t \to \infty$ in the sense of Radon measures.

Mathematics Subject Classification (2010): 35K40 (primary); 35B44, 35K61 (secondary).