

Gradient regularity for nonlinear parabolic equations

TUOMO KUUSI AND GIUSEPPE MINGIONE

To Emmanuele DiBenedetto on his 65th birthday

Abstract. We consider non-homogeneous degenerate and singular parabolic equations of the p -Laplacian type and prove pointwise bounds for the spatial gradient of solutions in terms of intrinsic parabolic potentials of the given datum. In particular, the main estimate found reproduces in a sharp way the behavior of the Barenblatt (fundamental) solution when applied to the basic model case of the evolutionary p -Laplacian equation with Dirac datum. Using these results as a starting point, we then give sufficient conditions to ensure that the gradient is continuous in terms of potentials; in turn these imply borderline cases of known parabolic results and the validity of well-known elliptic results whose extension to the parabolic case remained an open issue. As an intermediate result we prove the Hölder continuity of the gradient of solutions to possibly degenerate, homogeneous and quasilinear parabolic equations defined by general operators.

Mathematics Subject Classification (2010): 35K55 (primary); 31C45 (secondary).