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Boundedness of Global Solutions for Nonlinear Parabolic Equations Involving Gradient Blow-up Phenomena

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Abstract. We consider a one-dimensional semilinear parabolic equation with a gradient nonlinearity. We provide a complete classification of large time behavior of the classical solutions u: either the space derivative u_x blows up in finite time (with u itself remaining bounded), or u is global and converges in C^1 norm to the unique steady state.
The main difficulty is to prove C^1 boundedness of all global solutions. To do so, we explicitly compute a nontrivial Lyapunov functional by carrying out the method of Zelenyak. After deriving precise estimates on the solutions and on the Lyapunov functional, we proceed by contradiction by showing that any C^1 unbounded global solution should converge to a singular stationary solution, which does not exist.
As a consequence of our results, we exhibit the following interesting situation:
– the trajectories starting from some bounded set of initial data in C^1 describe an unbounded set, although each of them is individually bounded and converges to the same limit;
– the existence time T^* is not a continuous function of the initial data.

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