

On fundamental groups related to degeneratable surfaces: conjectures and examples

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Abstract. We argue that for a smooth surface S , considered as a ramified cover over \mathbb{CP}^2 , branched over a nodal-cuspidal curve $B \subset \mathbb{CP}^2$, one could use the structure of the fundamental group of the complement of the branch curve $\pi_1(\mathbb{CP}^2 - B)$ to understand other properties of the surface and its degeneration and vice-versa. In this paper, we look at embedded-degeneratable surfaces — a class of surfaces admitting a planar degeneration with a few combinatorial conditions imposed on its degeneration. We close a conjecture of Teicher on the virtual solvability of $\pi_1(\mathbb{CP}^2 - B)$ for these surfaces and present two new conjectures on the structure of this group, regarding non-embedded-degeneratable surfaces. We prove two theorems supporting our conjectures, and show that for $\mathbb{CP}^1 \times C_g$, where C_g is a curve of genus g , $\pi_1(\mathbb{CP}^2 - B)$ is a quotient of an Artin group associated to the degeneration.

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