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# Metric currents, differentiable structures, and Carnot groups

MARSHALL WILLIAMS

**Abstract.** We examine the theory of metric currents of Ambrosio and Kirchheim in the setting of spaces admitting differentiable structures in the sense of Cheeger and Keith. We prove that metric forms which vanish in the sense of Cheeger on a set must also vanish when paired with currents concentrated along that set. From this we deduce a generalization of the chain rule, and show that currents of absolutely continuous mass are given by integration against measurable $k$-vector fields. We further prove that if the underlying metric space is a Carnot group with its Carnot-Carathéodory distance, then every metric current $T$ satisfies $T\lfloor_{\theta} = 0$ and $T\lfloor_{d\theta} = 0$, whenever $\theta \in \Omega^1(\mathbb{G})$ annihilates the horizontal bundle of $\mathbb{G}$. Moreover, this condition is necessary and sufficient for a metric current with respect to the Riemannian metric to extend to one with respect to the Carnot-Carathéodory metric, provided the current either is locally normal, or has absolutely continuous mass.

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