Ann. Sc. Norm. Super. Pisa Cl. Sci. (5)
Vol. XI (2012), 215-241

On the definition and properties of p-harmonious functions

JUAN J. MANFREDI, MIKKO PARVIAINEN AND JULIO D. ROSSI

Abstract. We consider functions that satisfy the identity

$$u_\varepsilon(x) = \frac{\alpha}{2}\left\{\sup_{\overline{B}_\varepsilon(x)} u_\varepsilon + \inf_{\overline{B}_\varepsilon(x)} u_\varepsilon\right\} + \beta \fint_{B_\varepsilon(x)} u_\varepsilon \, dy$$

for a bounded domain in $\mathbb{R}^n$. Here $\varepsilon > 0$ and α, and β are suitable nonnegative coefficients such that $\alpha + \beta = 1$. In particular, we show that these functions are uniquely determined by their boundary values, approximate p-harmonic functions for $2 \leq p < \infty$ (for a choice of p that depends on α and β), and satisfy the strong comparison principle. We also analyze their relation to the theory of tug-of-war games with noise.

Mathematics Subject Classification (2010): 91A15 (primary); 35B50, 35J25, 35J70, 49N70, 91A24 (secondary).