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Two solutions for a singular elliptic equation by variational methods

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Abstract. We find two nontrivial solutions of the equation $-\Delta u = (-\frac{1}{u^\beta} + \lambda u^p)\chi_{\{u>0\}}$ in Ω with Dirichlet boundary condition, where $0 < \beta < 1$ and $0 < p < 1$. In the first approach we consider a sequence of ε-problems with $1/u^\beta$ replaced by $u^q/(u+\varepsilon)^{q+\beta}$ with $0 < q < p < 1$. When the parameter $\lambda > 0$ is large enough, we find two critical points of the corresponding ε-functional which, at the limit as $\varepsilon \to 0$, give rise to two distinct nonnegative solutions of the original problem. Another approach is based on perturbations of the domain Ω, we then find a unique positive solution for λ large enough. We derive gradient estimates to guarantee convergence of approximate solutions u_ε to a true solution u of the problem.

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