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Sharp Liouville results for fully nonlinear equations with power-growth nonlinearities

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Abstract. We study fully nonlinear elliptic equations such as

$$F(D^2u) = u^p, \quad p > 1,$$

in $\mathbb{R}^n$ or in exterior domains, where F is any uniformly elliptic, positively homogeneous operator. We show that there exists a critical exponent, depending on the homogeneity of the fundamental solution of F, that sharply characterizes the range of $p > 1$ for which there exist positive supersolutions or solutions in any exterior domain. Our result generalizes theorems of Bidaut-Véron [6] as well as Cutri and Leoni [11], who found critical exponents for supersolutions in the whole space $\mathbb{R}^n$, in case $-F$ is Laplace's operator and Pucci's operator, respectively. The arguments we present are new and rely only on the scaling properties of the equation and the maximum principle.

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