Maximizers for the Strichartz norm
for small solutions of mass-critical NLS

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Abstract. Consider the mass-critical nonlinear Schrödinger equations in both focusing and defocusing cases for initial data in L^2 in space dimension N. By Strichartz inequality, solutions to the corresponding linear problem belong to a global L^p space in the time and space variables, where $p = 2 + \frac{4}{N}$. In $1D$ and $2D$, the best constant for the Strichartz inequality was computed by D. Foschi who has also shown that the maximizers are the solutions with Gaussian initial data.

Solutions to the nonlinear problem with small initial data in L^2 are globally defined and belong to the same global L^p space. In this work we show that the maximum of the L^p norm is attained for a given small mass. In addition, in $1D$ and $2D$, we show that the maximizer is unique and obtain a precise estimate of the maximum. In order to prove this we show that the maximum for the linear problem in $1D$ and $2D$ is nondegenerated.

Mathematics Subject Classification (2010): 35Q55 (primary); 35P25, 35B50, 35B45 (secondary).